A Circadian Rhythms Learning Network for Resisting Cognitive Periodic Noises of Time-Varying Dynamic System and Applications to Robots

Zhijun Zhang*, Senior Member, IEEE, Xianzhi Deng, Lingdong Kong, Student Member, IEEE and Shuai Li, Senior Member, IEEE

Abstract—Time-varying dynamic system contaminated by cognitive noises is universal in the fields of engineering and science. In this paper, a circadian rhythms learning network (termed CRLN) is proposed and investigated for disposing noise disturbed time-varying dynamic system. To do so, a vector-error function is firstly defined. Secondly, a neural dynamic model is formulated. Thirdly, a co-state matrix is integrated into the model, of which the states are the linear combination of the previous periodic states and errors, which can effectively suppress periodic noises. Theoretical analysis and mathematical derivation prove the global exponential convergence performance of the proposed CRLN model. Finally, a practical noise disturbed time-varying dynamic system example with four different noises illustrates the accuracy and efficacy of the proposed CRLN model. Comparisons with traditional zeroing neural network further verify the advantages of the proposed CRLN model.

Index Terms—Circadian rhythms, neural network, time-varying problem, large errors, online equation solving, convergence.

I. INTRODUCTION

DYNAMIC systems play an important role in science and engineering fields, such as machine learning control theory [1], [2], and robotics [3], [4]. Most dynamic system can be transformed into a matrix equation solving problem i.e., linear least-squares regression [5], optimal controller design [6], robot motion planning [7] and so on. The traditional approach is the numerical methods [8], [9], of which the minimal arithmetic operations are proportional to the cubic of matrix dimension [10]. In addition, most of the numerical methods were exploited to solve the time invariant problems and did not consider the large noises. Therefore, it is prohibitively expensive or even impossible to apply to a time-varying dynamic system problems using the traditional numerical methods.

With the development of the parallel nature and convenience of electronic implementation [11], [12], some parallel-processing models, such as neural networks, have been designed to break through the bottleneck of the traditional approaches [10]–[19]. Compared with the traditional numerical mathematics methods, recurrent neural networks possess various advantages. In addition to superiorities of disturbed storage, adaptive self-learning capability and parallelism, recurrent neural networks also have advantages of simplicity for implementation on hardware [15], [16]. These kinds of dynamic and analog solvers based on neural networks are able to obtain high accuracy and fast convergent rate for handling with online time-varying dynamic system [11], [13], [14]. For instance, in Ref. [20], a classic neural network based on gradient method was proposed for the real-time computation of semidefinite programming. The gradient neural network used a scalar-valued cost function, and its minimum point is the solution of the matrix equation problem. The state update process evolves along a descent direction of this cost function until the minimum is obtained.

However, majority of the neural networks mentioned above were designed intrinsically to solve static problems with fixed parameters. Actually, quantities of systems are inherently dynamic, which utilize time-varying parameters. Recently, online time-varying dynamic systems processing is considered as essential problem in engineering applications and scientific research fields, such as signal processing [21], [22], image fusion [23] and so on. The gradient neural networks can only work approximately, which may be not qualified for time-varying solving. Therefore, in order to obtain a more accurate solution in time-varying computation process, more efficient neural networks are necessary. In order to obtain more accurate values, Zhang et al. proposed an implicit dynamics method, called zeroing neural network, for solving time-varying problems [24]. The traditional zeroing neural network (termed TZNN) was designed by defining a matrix-valued error-monitoring function, and used a first-order time derivatives to bring in some prediction capacities. It utilizes
the time derivative form of error function to accurately track the time-varying solution of system. With these designing architecture, the zeroing neural network can achieve global exponential convergence. The detailed comparisons between the zeroing neural network and gradient-based neural network can be seen from Ref. [25]. In order to improve the computation performance, Xiao et al. proposed a finite-time convergent neural dynamics for solving a time-varying linear complex matrix equation problem [26].

Nevertheless, the ideal noise-free time-varying dynamic systems seldom exist in the actual hardware implementation. The noise usually exists in network system and has effect on the system. For instance, Zhu et al. revealed that strong common noises may destroy the function of neurons [27]. Shi et al. demonstrated that noise disturbance degrades the cooperative coupling strength inducted synchronization transitions [28]. Zhao et al. investigated the influence of noise intensity in weak electric signals detect fields [29]. Hence, how to eliminate the impact of noise interference is a vital issue about the stabilities of systems. Eile et al. proposed a novel nonlinear controller electronic circuit design method to restrain the noise effects on robust synchronization of a small pacemaker neuronal ensemble [30]. In [31], Wang et al. proposed an approach to evaluate the range of noise intensity to further suppress the noise influence in membrane potential of neurons. In [32], Yao et al. proposed a weak periodic signal detection approach in the FitzHugh-Nagumo neuron to restrain the periodic signal noise impact.

Specifically, periodic noise is frequently arises in various engineering and science fields and degrades the performance of the systems, i.e., the coggging effect of the permanent magnet is regraded as the periodic noise, which severely influences the servo control performance in low-speed control systems [33]. The friction force is a typical periodic noise in low velocities control and the noise is particularly conspicuous during low velocities motion control [34]. In [35], the authors revealed that some respiratory equipment, i.e., continuous positive airway pressure devices will create constant noise. Besides, the broad-band noise in MOSFET can be regarded as square wave periodic noise [36]. The audible noise caused by transmission, recording and quantization of audio signals is a type of sawtooth periodic noise [37]. Additionally, in the course of seismic exploration, sinusoidal noise is a tough influence of the analysis of seismic records [38]. The friction and eccentricity of the wheeled mobile robots is another kind of periodic noise [39]. It is worth pointing out that gaussian white noise commonly exists in natural environment and manmade source such as the atmospheric noise detected by the sensors and electrically conductive components in the devices. Hence in some cases, gaussian white noise can combine with periodic noise to influence the performance of systems, which can be also approximatively viewed as periodic noise [40]. Since these periodic noises can be detected by the sensors, we generally named it cognitive periodic noise.

Since the periodic noises have nonnegligible influence on the accuracy of neural networks solution, and in some cases, these noises are able to ruin the whole solving process. Hence, various denoising preprocessing approaches have been proposed to reduce the influence of noise. Zhao et al. proposed an improved threshold denoising method with wavelet transform to improve the quality of the polluted signal [41]. A novel denoising approach based on variational mode decomposition was proposed for fault feature extraction in [42]. In [43], Wei et al. utilized singular value decomposition to design an improved ensemble empirical mode decomposition denoising method for disposing the chaotic signal. According to Bregman iterative regularization and gradient projection algorithms, Tong et al. proposed a weighted denoising approach to resist noise in digital image process [44]. Worth mentioning, the aforementioned noise suppressing methods are defined as preprocessing for noise, which will consume extra time cost for disposing noise. Therefore the preprocessing is against the time economy principle and is not appropriate for time-critical dynamic systems. Hence, it is preferable to integrate denoising with solving process in a parallel manner.

Owing to the parallel computational and powerful noise tolerate properties, numerous neural networks have been investigated for suppressing periodic noise and avoiding extra noise preprocessing time in computational process. For instance, Li et al. proposed a novel modified primal-dual neural network to restrain a kind of periodic noise, the harmonic disturbances in the input channel, which generates in the control process of redundant manipulators [45]. In [46], Zhang et al. designed a novel recurrent neural network to suppress the input periodic disturbance for kinematic control of redundant manipulators. However, the aforementioned methods can only apply to several particular form of periodic noise, which is not capable of applying to the general form of periodic noise.

Additionally, the disturbance of time delay is an important problem, which usually originates from the communication time of finite switching speed amplifiers [47]. Specifically, in the neural network fields, time delay will degrade the performance of neural networks, and may even cause some unexpected negative effects, i.e., instability, chaos, divergence, oscillation or other destructive influence [48]–[51]. Worth mentioning, our proposed CRLN can be applied to time delay systems as well.

Hence, in order to dispose the time-varying dynamic system online with resistance to cognitive periodic large noises, a circadian rhythms learning network is proposed in this paper. This novel neural network can not only deal with time-varying dynamic system, but also eliminate the cognitive periodic noises. It does not need extra time and computation consuming, and has efficient computing power. Besides, the proposed neural network is capable of restraining the general form of periodic noise. Besides, since the practical systems usually have a time delay [52], which will degrade the performance of the systems, the proposed CRLN can be applied to time delay systems as well.

The reminder of this paper is organized into five sections. Section II gives the problem formulation and the circadian rhythms learning network (CRLN) is proposed. For comparison, the traditional zeroing neural network is also introduced in this section. Theoretical analysis and mathematics proofs are presented in detail in Section III. Section IV illustrate four examples to show the superiority and efficacy of the proposed
CRLN model. The application of circadian rhythms learning network to robotic kinematics control system is shown in Section V. Section VI concludes this paper with final remarks. Before ending this section, the main contributions of this paper are summarized as the following facts:

- A novel circadian rhythms learning network (CRLN) is proposed to resist cognitive periodic noises in time-varying dynamic system. To the best of author’s knowledge, it is the first time to propose this kind of neural network model for cognitive periodic noise disturbed time-varying system processing.

- Exponential convergence properties of CRLN with resistance to cognitive periodic noises are discussed and analyzed in detail, which guarantees that the proposed CRLN model is able to globally converge to the exact theoretical real-time solution of time-varying dynamic system in an exponential convergence rate.

- Compared with traditional zeroing neural network, the proposed CRLN with cognitive periodic time-varying parameter achieves a better accuracy performance and can well eliminate the cognitive periodic noises in computation.

- Computer simulation results of time-varying matrix equation example are illustrated in detail to substantiate the excellent cognitive periodic noises suppression property of the proposed CRLN.

II. Problem Formulation and Neural Network Design

In this section, the proposed CRLN model is designed in detail, and for comparisons, the traditional zeroing neural network is also presented.

A. Time-Varying Matrix Equation Problem

Consider the time-varying dynamic system in the mathematical form of

\[ A(t)X(t) = B(t), \quad \forall t \in [0, +\infty) \]  

(1)

where \( t \) denotes the time variable, \( A(t) \in \mathbb{R}^{m \times m} \) and \( B(t) \in \mathbb{R}^{m \times n} \) are time-varying coefficient matrices, and \( X(t) \in \mathbb{R}^{n \times n} \) denotes the unknown matrix to be obtained. Additionally, the time-varying matrices \( A(t) \), \( B(t) \), as well as their time derivative matrices \( \dot{A}(t) = \frac{dA(t)}{dt}, \dot{B}(t) = \frac{dB(t)}{dt} \), are smoothly time-varying and assumed to be known or can be precisely estimated. The goal of the proposed CRLN method is to obtain the optimal system solution of Equation (4) for \( X(t) \in \mathbb{R}^{m \times n} \) in real time with multiple kinds of cognitive periodic large noises.

B. Circadian Rhythms Learning Network

In order to solve Equation (4) with large period noises, a circadian rhythms learning network (CRLN) can be obtained via the following design process.

1) Firstly, a matrix-valued error function related to Equation (4) is defined to monitor the computing process, i.e.,

\[ E(t) = A(t)X(t) - B(t). \]  

(2)

2) Secondly, in order to make each entry \( e_{ij} \) with \( i = 1, \ldots, m, j = 1, \ldots, n \) of the above error function \( E(t) \in \mathbb{R}^{m \times n} \) converge to zero, a descent direction is expected. Considering the cognitive periodic noises, the implicit dynamics of Equation (2) is designed as

\[ \dot{E}(t) = -\alpha F(E(t)) - \lambda(t) + \Delta \omega(t) \]  

(3)

where \( \dot{E}(t) = \frac{dE(t)}{dt}; \alpha > 0 \) is the design parameter to scale the convergence rate of the CRLN; \( \Delta \omega(t) \in \mathbb{R}^{m \times n} \) denotes the cognitive periodic noise with unknown amplitudes. The periodic noise can be any type of periodic function, i.e., the square wave noise, the triangular wave noise, the cognitive periodic noise and so on. It is worth pointing out that the constant noise can be considered as a special type noise. We take the general form of the periodic noise into consideration. Utilizing the technology of Fourier series, the general periodic noise can be presented as

\[ \Delta W(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \]  

(4)

\[ a_k = \frac{1}{T} \int_T \Delta W(t)e^{-jk\omega_0 t}dt \]

where \( a_k \) denotes the Fourier coefficient, \( \omega_0 \) denotes the noise frequency and \( T \) denotes the period of periodic noise. Worth mentioning, this general form is capable of representing the actual practical noise in realtime implementation with different Fourier coefficient, including the coggling effect in [33] and the friction in [34]. In this paper, four kinds of periodic noise are used to test CRLN model, i.e.,

- Constant noise:

![Block diagram of CRLN model for time-varying matrix equation solving.](image-url)
where $\rho$ represents the amplitude of constant noise.

- Square wave noise:

$$
\Delta W_2(t) = \begin{cases} 
0 & \text{for } T - T/2 < t \leq T - T_1 \\
\tau & \text{for } T - T_1 < t \leq T_1 + kT \\
0 & \text{for } T_1 + kT < t \leq T/2 + kT 
\end{cases}
$$

$$
\sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t}
$$

where $\tau$ denotes the amplitude of square wave noise, $T_1$ denotes the time period of $\tau$ amplitude, $k$ denotes positive integer and $T$ represents the period of square wave noise.

- Triangular wave noise:

$$
\Delta W_3(t) = \frac{\sigma}{T} \cdot (\text{mod}(t, T) - T/2) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t}
$$

$$
a_k = \begin{cases} 
\frac{\sigma}{2\pi k} (-1)^k j & k > 0 \\
\frac{\sigma}{2\pi k} (-1)^{k+1} j & k < 0 \\
0 & k = 0
\end{cases}
$$

where $\sigma$ denotes the slope of the sawtooth noise, $T$ denotes the period of sawtooth noise and $\text{mod}(t, T)$ denotes the remainder of $t$ divided by $T$.

- Sinusoidal noise:

$$
\Delta W_4(t) = \sin(\omega_0 t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t}
$$

$$
a_k = \begin{cases} 
\frac{\sigma_k}{2\pi j} & k = 1 \\
\frac{\sigma_k}{-2\pi j} & k = -1 \\
0 & k \neq \pm 1
\end{cases}
$$

where $\omega_0$ denotes the frequency of the periodic noise and $\rho$ denotes the amplitude of the sinusoidal noise.

$\lambda(t) \in \mathbb{R}^{m \times n}$ denotes the co-state matrix, and

$$
\lambda(t) = \begin{cases} 
\lambda(t - T) + \beta E(t) & t > T \\
0_{n \times m} & t \leq 0
\end{cases}
$$

where design parameter $\beta > 0$ denotes the feedback factor, and parameter $T$ denotes the period of noise $\Delta \omega(t)$. Parameter $\alpha > 0$ is designed to guarantee the convergence rate, and $\mathcal{F}(\cdot) : \mathbb{R}^{m \times n} \to \mathbb{R}^{m \times n}$ denotes the matrix-valued activation function mapping. According to above implicit dynamic equation, block diagram realization of CRLN is designed as displayed in Fig. 1. The block diagram is the illustration of Equation (3) as well as the realization of the proposed CRLN model. It is worth pointing out that CRLN can be realized by adopting some electronic components. Additionally, the block diagram is capable of contributing to the design procedure of CRLN for physical implementation. In Fig. 1, $\sum$ denotes the accumulator and $\int$ denotes the integrator. In this paper, four kinds of activation functions are used to test CRLN model, i.e.,

- Linear activation function:

$$
\mathcal{F}_1(u) = u
$$

where $u$ is a real scalar or a matrix. Variable $u$ has the same definition in the following situations.

- Power-sigmoid activation function:

$$
\mathcal{F}_2(u) = \begin{cases} 
\frac{u^p}{1 + \exp(-\xi u)} & \text{if } |u| \geq 1 \\
\frac{\xi u}{1 + \exp(-\xi u)} & \text{otherwise}
\end{cases}
$$

where $p \geq 2$, $p$ is odd and $\xi \geq 3$.

- Sinh activation function:

$$
\mathcal{F}_3(u) = \frac{\exp(u) - \exp(-u)}{2}
$$

- Tunable activation function:

$$
\mathcal{F}_4(u) = \sigma_r(u) + \sigma(u) + \sigma^T(u)
$$

where $r > 0$ and $r \neq 1$.

Function $\sigma_r(u)$ is defined as

$$
\sigma_r(u) = \begin{cases} 
|u|^r, & \text{if } u > 0 \\
0, & \text{if and only if } u = 0 \\
-|u|^r, & \text{if } u < 0
\end{cases}
$$

where $|u|$ denotes the absolute value of $u \in \mathbb{R}$. In addition, the activation functions $\mathcal{F}(u)$ should be odd and monotonically increasing.
3) Thirdly, Equation (3) can be reformulated as the following implicit dynamics, i.e.,
\[ A(t)X(t) = - \dot{A}(t)X(t) + \dot{B}(t) + \Delta \omega(t) - \lambda(t) \]
\[ - \alpha F(A(t)X(t) - B(t)). \]  
(8)

When using linear activation function \( F_I(u) = u \), the matrix equation with noise (3) can be simplified as
\[ A(t)X(t) = - \dot{A}(t)X(t) + \dot{B}(t) + \Delta \omega(t) - \lambda(t) \]
\[ - \alpha (A(t)X(t) - B(t)). \]  
(9)

Remark 1. Since continuous control laws cannot be imposed in the implementation of the practical systems, the discretization form of proposed CRLN should be investigated. Utilizing the technology of Taylor series, we can obtain the Taylor-type numerical differentiation formula as follows
\[ \dot{g}(t_k) \approx \frac{2g(t_{k+1}) - 3g(t_k) + 2g(t_{k-1}) - g(t_{k-2})}{2h} \]  
(10)

where \( g(.) \) represents all the functions whose first three derivatives with itself are continuous on the range of hypothetical interval \([a, b] \). Meanwhile, the time instant \( t_{k-2, k-1, k, k+1} \in [a, b] \). It is worth mentioning that above Taylor-type numerical differentiation formula has a truncation error of \( O(h^3) \). Hence, based on Taylor-type numerical differentiation formula, we can obtain the discretization form of proposed CRLN with omitting the truncation error as:
\[ A_k(X_{k+1} - 1.5X_k + X_{k-1} - 0.5X_{k-2}) = \]
\[ h(-\dot{A_k}X_k + \dot{B_k} + \Delta \omega_k - \lambda_k - \alpha F(A_kX_k - B_k)). \]  
(11)

where \( A_k, B_k, \lambda_k, \) and \( \Delta \omega_k \) respectively represent the value of each different matrix when \( t = t_k \), \( X_k, X_{k-1} \) and \( X_{k-2} \) relatively denote the value of \( X(t) \) when \( t = t_{k+1}, t = t_k, t = t_{k-1} \) and \( t_{k-2} \).

Remark 2. Based on the definition of time complexity, the time complexity of the proposed neural network is \( O(m^2n) \) when using the linear activation function. Besides, the complexity of the scale of the network circuits is presented as follows, which can be measured by the numbers of the processor units. Assume that the maximum number of iterations of each computation is \( \mu \), then Circadian Rhythms Learning Network contains \( mn \) integral operators, \( (\mu + 2)m^2n + mn \) summers, \( (\mu + 2)m^2n \) multipliers, \( 2m^2n^2 \) differential operators and a \( mn \) dimension activation function \( F(.) \).

Different from the traditional approach adding a filter to reduce the noise, the proposed CRLN method does not need any preprocessing and extra time consuming operation. It is very suitable for time-varying problem solving and applied to real-time computation. The proposed CRLN model (8) is capable of restraining various kinds of noises and computing the time-varying dynamic system (4) simultaneously. Compared with the traditional zeroing neural network, the proposed CRLN model has more powerful period noise suppression ability.

C. Traditional Zeroing Neural Network

For comparisons and for illustration, the traditional zeroing neural network [14], [53]–[55] is presented in this section.

Firstly, an error function is defined as \( E(t) = A(t)X(t) - B(t) \).

Secondly, the differential form of error function \( E(t) \) with cognitive periodic noises is \( dE(t)/dt = -\alpha F(E(t)) + \Delta \omega(t) \), where \( \alpha > 0 \in \mathbb{R} \) is used to command the convergence rate.

With the above two steps, the traditional zeroing neural network can be designed as
\[ A(t)X(t) = - \dot{A}(t)X(t) + \dot{B}(t) + \Delta \omega(t) \]
\[ - \alpha_2 F(A(t)X(t) - B(t)). \]  
(12)

where \( F(.) \) denotes the activation function, and \( \alpha_2 \) is a fixed-parameter designed to obtain better convergence speed.

III. THEORETICAL ANALYSIS

In this section, a noise suppression theorem is proposed and proved in this section.

Theorem 1. Given matrix functions \( A(t) \in \mathbb{R}^{m \times n} \) and \( B(t) \in \mathbb{R}^{m \times m} \), if a time-varying dynamic system (4) is solved by a circadian rhythms learning network (3) with the periodic vector-form noise \( \Delta \omega(t) = \Delta \omega(t - T) \), starting from any initial state \( X(0) \in \mathbb{R}^{m \times m} \), the state solutions generated from the neural network are able to globally converge to the theoretical solution \( X^*(t) \) of (4).

Proof: The design formula of CRLN \( \dot{E}(t) = -\alpha F(E(t)) - \lambda(t) + \Delta \omega(t) \) is regarded as a compact matrix-form of following \( m \times n \) equations
\[ \dot{E}_{ij}(t) = -\alpha F(E_{ij}(t)) - \lambda_{ij}(t) + \Delta \omega_{ij}(t) \]  
(13)

where \( E_{ij}(t) \) denotes the \( ij \)th element of \( E(t) \) with \( i = 1, \ldots, m \) and \( j = 1, \ldots, n \).

In order to prove that the scalar-type CRLN (13) can suppress noises, an intermediate variable \( \tilde{\omega}(t) \) is defined as
\[ \tilde{\omega}(t) = -\Delta \omega(t) + \lambda(t). \]  
(14)

Since the noise \( \Delta \omega(t) \) is the period noise, i.e.,
\[ \Delta \omega(t) = \Delta \omega(t - T). \]  
(15)

Based on Equation (6) and Equation (15), the intermediate variable \( \tilde{\omega}(t) \), i.e., Equation (14), can be reformulated as
\[ \tilde{\omega}(t) = \tilde{\omega}(t - T) + \beta E(t). \]  
(16)

Utilizing linear activation function \( F_I(u) = u \) and according to the definition of \( \tilde{\omega}(t) \), Equation (13) can be reformulated as
\[ \dot{E}_{ij}(t) = -\alpha E_{ij}(t) - \tilde{\omega}_{ij}(t). \]  
(17)

It follows from the Laplace Transformation [56] that Equation (16) can be reformulated as
\[ \tilde{\omega}_{ij}(s) = \tilde{\omega}_{ij}(s)e^{-sT} + \beta E_{ij}(s) \]  
(18)

where \( \tilde{\omega}_{ij}(s)e^{-sT} \) is the Laplace transformation of \( \tilde{\omega}_{ij}(t - T) \) and Equation (17) is
\[ sE_{ij}(s) - E_{ij}(0) = -\alpha E_{ij}(s) - \tilde{\omega}_{ij}(s). \]  
(19)
Substituting Equation (18) into Equation (19), $E_{ij}(s)$ is obtained as
\[
E_{ij}(s) = (s + \alpha + \frac{\beta}{1 - e^{-sT}})^{-1} E_{ij}(0). \tag{20}
\]

According to the theorem of stability [57], [58], the system is expected to remain stable if and only if the roots of the characteristic equation
\[
s + \alpha + \frac{\beta}{1 - e^{-sT}} = 0 \tag{21}
\]
have to fall in the left-half plane (the real part of the root should be non-positive).

Characteristic equation (21) can be reformulated as
\[
e^{-sT} = 1 + \frac{\beta}{s + \alpha}. \tag{22}
\]
Since $s$ is a complex number, and we can define $s = a + bj$. Equation (22) is further rewritten as
\[
e^{-aT}e^{-bjT} = \frac{a + \alpha + \beta + bj}{a + \alpha + bj} = \frac{(a + \alpha)^2 + \beta(a + \alpha) + b^2 - b\beta j}{(a + \alpha)^2 + b^2}. \tag{23}
\]

According to the Euler formula, we can obtain
\[
e^{-bjT} = \cos(-bT) + j\sin(-bT). \tag{24}
\]
Therefore, the real part of Equation (23) is
\[
e^{-aT}\cos(-bT) = \frac{(a + \alpha)^2 + b^2 + \beta(a + \alpha)}{(a + \alpha)^2 + b^2}. \tag{25}
\]
We assume that the system is not stable, i.e., the real part $a$ of roots $s$ of the characteristic equation (21) is nonnegative. That is $a \geq 0$. Since $a \geq 0$, $T > 0$, $\cos(-bT) \leq 1$, the left side of Equation (25) is not larger than 1, i.e.,
\[
e^{-aT}\cos(-bT) \leq 1. \tag{26}
\]

Considering the right side of Equation (25), we can obtain following equation, i.e.,
\[
\frac{(a + \alpha)^2 + b^2 + \beta(a + \alpha)}{(a + \alpha)^2 + b^2} = 1 + \frac{\beta(a + \alpha)}{(a + \alpha)^2 + b^2}. \tag{27}
\]
Considering the parameters $a$, $\alpha$ and $\beta$ are all positive, Equation (27) is larger than 1, i.e.,
\[
\frac{(a + \alpha)^2 + b^2 + \beta(a + \alpha)}{(a + \alpha)^2 + b^2} > 1. \tag{28}
\]

IV. ILLUSTRATIVE EXAMPLES

In this section, comparative simulations are conducted to verify the robustness of the proposed CRLN model (3). For comparisons, simulative results of traditional zeroing neural network (12) are also presented to illustrate the superiority of the proposed CRLN model (3) when solving a time-varying matrix equation (4) with cognitive periodic noises. The simulation is performed with MATLAB R2017a, on a Lenovo ideapad700 with Intel Core i5 CPU at 2.30GHz, 8.00GB of RAM.

Consider the following real-time solution of time-varying dynamic system $A(t)X(t) = B(t)$ (4) with coefficient matrices $A(t) \in \mathbb{R}^{m \times m}$, $B(t) \in \mathbb{R}^{m \times 1}$, i.e.,
\[
A(t) := \begin{bmatrix}
\cos(t) & \sin(t) \\
-\sin(t) & \cos(t)
\end{bmatrix},
\]
\[
B^T(t) := [2\cos(t)\sin(t) \cos^2(t) - \sin^2(t)].
\]
The time derivatives of the above coefficient matrices are
\[
\dot{A}(t) := \begin{bmatrix}
-\sin(t) & \cos(t) \\
-\cos(t) & -\sin(t)
\end{bmatrix},
\]
\[
\dot{B}(t) := \begin{bmatrix}
2\cos(t)\sin(t) & \cos^2(t) - \sin^2(t)
\end{bmatrix}.
\]

![Fig. 3. Four Different kinds of noises. (a) Constant noise. (b) Square wave noise. (c) Triangular wave noise. (d) Random periodic noise.]

TABLE I

<table>
<thead>
<tr>
<th>Activation function</th>
<th>CRLN</th>
<th>The average time of each iteration</th>
<th>Computation time cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>4.95 x 10^{-3}</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Power-Sigmoid</td>
<td>4.67 x 10^{-5}</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Sinh</td>
<td>4.18 x 10^{-5}</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Tunable</td>
<td>5.11 x 10^{-5}</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>
investigate the constant noise situation for simplicity. It is worth pointing out that the constant noise can be seen as a special type of the cognitive periodic noise, and we first consider the following equation:

$$\dot{B}(t) := \begin{bmatrix} 2 \cos^2(t) - 2 \sin^2(t) & -4 \cos(t) \sin(t) \end{bmatrix}.$$  \hspace{1cm} (29)

The theoretical solution $X^*(t)$ with the above coefficients to the time-varying system $A(t)X(t) = B(t)$ is

$$X^*(t) := \begin{bmatrix} \sin(t) & \cos(t) \end{bmatrix}.$$  

A. Comparisons with Traditional Zeoring Neural Network

In the following section, four different types of noises are used to test the anti-noise-interference ability of the proposed CRLN (3), i.e., the constant noise, square wave noise, triangular wave noise and random periodic noises. For comparisons, we also present the simulative results of the traditional zeoring neural network (12) when it is used to solve Equation (4). It is worth pointing out that the constant noise can be seen as a special type of the cognitive periodic noise, and we first investigate the constant noise situation for simplicity.

1) Constant Noise. In actual implementation process, when the system is disturbed by a constant noise, such as a constant model error, the desired object could be difficult to track. This constant noise can also affect the convergence performance of computation models and the computation accuracy would be rapidly deteriorating. Specifically, the authors revealed that some respiratory equipment, i.e., continuous positive airway pressure devices will create constant noise [35]. In the CRLN (3) and TZNN (12), $\alpha = 10$ and $\beta = 1000$. In addition, the parameters $p$ and $\xi$ in power-sigmoid activation function $F_2$ are $p = 3$ and $\xi = 4$, respectively. When using the tunable activation function $F_4$, the parameter $r = 0.5$. The simulation results show that, by using CRLN model (3), the neural networks with a power-sigmoid $F_2$, a sinh $F_3$ or a tunable $F_4$
different activation functions in a computational system. (b) Computational errors with a power-sigmoid activation function. (c) Computational errors with a sinh activation function.

Fig. 6. State solution $X(t)$ generated by CRLN model (3) (blue solid line) with parameters $\alpha = 1$ and $\beta = 100$ and TZNN model (12) (red dash line) with different activation functions in $T \in [0, 5]$ and the corresponding theoretical solution $X^*(t)$ (black dot line) when they are applied to dispose time-varying dynamic system (4) with the square wave noise. (a) Solutions generated by CRLN and TZNN models with a linear activation function. (b) Solutions generated by CRLN and TZNN models with a power-sigmoid activation function. (c) Solutions generated by CRLN and TZNN models with a sinh activation function. (d) Solutions generated by CRLN and TZNN models with a tunable activation function.

Fig. 7. Comparisons of computational errors $||A(t)X(t) - B(t)||_F$ between CRLN model (3) with parameters $\alpha = 1$ and $\beta = 100$ and TZNN model (12) with different activation functions in $T \in [0, 5]$ when dealing with time-varying dynamic system (4) with the square wave noise. (a) Computational errors with a linear activation function. (b) Computational errors with a power-sigmoid activation function. (c) Computational errors with a sinh activation function. (d) Computational errors with a tunable activation function.

| Activation function | CRLN
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<td>Sinh</td>
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<td>Tunable</td>
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Table II: The computation time (s) between CRLN model (3) with parameters $\alpha = 10$ and $\beta = 1000$ with different activation functions when dealing with time-varying dynamic system (4) with the square wave periodic noise

Activation functions have better convergence performance than the network with a linear activation function $F_1$. In a word, the proposed CRLN model (3) can effectively repress the constant noise. In addition, the better performance can be obtained by selecting appropriate activation function.

2) Square Wave Noise. Actually, square wave periodic noises usually arise in the fields of electronic devices, i.e., the broad-band noise in MOSFET can be regarded as square wave periodic noise [36]. In order to further test the general applicability, square wave noises $\Delta \omega$ are added in the system. The parameter of two neural networks and the four activation functions are the same as the previous simulations.

The corresponding simulation results are shown in Figs. 6 and 7. We can see from Fig. 6 that, starting from any randomly generated initial state, if the CRLN model (3) is applied, the computing error $||A(t)X(t) - B(t)||_F$ would rapidly converge towards zero in 5 s. By contrast, the residual error of TZNN model (12) has a decreasing trend at the beginning, but as time evolves, the error rebounds and remains unstable. Besides, the computing error of TZNN model (12) is nearly 100 times larger than that of CRLN no matter what activation functions are used. This result further illustrates the superiority of CRLN model (3) for resisting the square wave noise in time-varying dynamic system. Similarly, Fig. 7 shows that the solutions generated by CRLN model (3) can converge to the theoretical solution very well while the online solutions of TZNN model (12) have deviated from the theoretical solutions $X^*(t)$. Additionally, Table II also demonstrates the superiority of CRLN in computation process at the presence of square wave noise.

3) Triangular Wave Noise. The audible noise caused by transmission, recording and quantization of audio signals is a type of triangular periodic noise [37]. In this subsection, the comparative simulations under the disturbance of triangular wave noise are conducted. The simulation results are shown in Figs. 8 and 9. It can be seen from Fig. 8 that the stability of CRLN model (3) is superior to TZNN model (12) with four different activation functions and the same parameter $\alpha = 10$. In addition, although the errors of TZNN model (12) decreases at the beginning, it can not converge to zero and the residual errors always exist, which implies $X(t)$
4) Random Periodic Noise. In this section, we take a more complicated form of periodic noise into consideration. Gaussian white noise commonly exists in natural environment and manmade source such as the data acquisition equipment in digital images or communication system and electrically conductive components in the devices. Hence, gaussian white noise can combined with periodic noise to influence the systems’ performance and may misdirect the model. Since the value of gaussian white noise at any pair of time is identically distributed and statistically independent, we select the sinusoidal periodic noise to gaussian white noise ratio as 1.5 \text{ db} to satisfy the amplitude of mixed noise is random at any time instant and remains the periodic property on the whole. Hence, we regarded this kind of mixed noise as random periodic noise. Fig. 3d presents the specific form of random periodic noise. Worth mentioning, the period of random periodic noise is viewed as 1s, for the period of sinusoidal noise is set as 1s.

Similar to the previous discussion, the convergence performance of TZNN model (12) and CRLN model (3) are compared and the simulation results about random periodic noise are shown in Figs. 10 and 11. From these two figures, we see that using whatever activation function, the solutions generated from CRLN model can converge...
to the theoretical solutions, and the time cost of CRLN model (3) when computational error converges to zero is very short. Specifically, the computational error rapidly diminishes to zero in $T \in [0, 5]$, and some neural network with tunable activation functions $F_A$ even can converge to zero in less than 2 seconds. By contrast, when using TZNN model (12) with the linear, power-sigmoid sinh and tunable activation functions, the computational error cannot converge to zero in $T \in [0, 5]$ and the error remains a relatively high level. Additionally, the concrete performance differs from different kinds of activation functions. As the simulation example shows that the power-sigmoid and tunable activation functions perform better than linear and sinh activation functions. Besides, as can be seen from Table IV, the total computational time is larger than previous three simple periodic noise simulations. Nevertheless, the proposed CRLN still processes superior convergence property to restrain this kind of complicated periodic noise.

In summary, the above simulations verify that the CRLN model (3) has the superior capacity to resist the cognitive periodic noise in time-varying dynamic system. In addition, the convergence performance is affected by different activation functions.

B. Parameter Comparisons

In order to further investigate the performance of the CRLN model (3) with different design parameters $\alpha$ and $\beta$, this section will show the simulation results with different design parameters $\alpha$ and $\beta$. We adapt four kinds of activation functions and consider the situation in the disturbance of square wave noise. Firstly, we consider parameter $\alpha$. The convergence time of different parameter $\alpha$ when computational error reach 0.005 with four kinds of activation functions and $\beta = 100$ is shown in Table V. From Table V, we know that no matter what activation functions are used, the convergence time demonstrates the declining tendency when parameter $\alpha$ is changing from 0.1 to 50. Similarly, the increasing of parameter $\beta$ can also improve the convergence performance (as shown in Table VI). This comparison result shows that the design parameter $\alpha$ and $\beta$ should be set as large as possible if the actual hardware permits for the better convergence performance.

V. APPLICATION TO ROBOT KINEMATICS

In this section, the proposed CRLN model (3) is applied to a specific time-varying dynamic system which is the motion planning of a six-link Kinova manipulator in
the cognitive periodic noise environment. Consider a six-link planar robot kinematics with joint-angle vector \( \theta(t) = [\theta_1(t), \theta_2(t), \theta_3(t), \theta_4(t), \theta_5(t), \theta_6(t)]^T \in \mathbb{R}^6 \), where \( \theta_i(t) \) denotes the \( i \)th joint angle. In order to demonstrate the superior robustness performance of the proposed CRLN model (3), the external disturbance and the digital computational error are taken into consideration.

According to the kinematics theory, it is difficult to obtain the solution to inverse-kinematic problem \( \theta(t) = f^{-1}(r(t)) \) at position level, where \( \theta(t) \) denotes the joint angle, \( f(\cdot) \) denotes a nonlinear mapping from joint-angular space to task space, and \( r(t) \) denotes the end-effector task. Since the nonlinearity of \( f(\cdot) \), it is difficult to obtain the inverse solution \( \theta^*(t) \). A general approach is to solve it at the joint-velocity level, and the forward kinematic equation at the velocity level is

\[
J(\theta(t)) \dot{\theta}(t) = \dot{r}(t)
\]

where \( \dot{r}(t) \) denotes the end-effector velocity and \( J(\theta(t)) \) is a Jacobian matrix equation defined as \( J = \partial f(\theta)/\partial \theta \). Hence, based on Equation (30), the solution to the inverse-kinematic problem can be obtained analytically, i.e.,

\[
\dot{\theta}(t) = J^{-1}(\theta(t)) \dot{r}(t)
\]

where \( J^{-1}(\theta(t)) \) denotes the inverse matrix of \( J(\theta(t)) \). Evidently, for the proposed of solving inverse-kinematic problem, obtaining \( J^{-1}(\theta(t)) \) in real time instant \( t \) is necessary. Actually, the inverse of Jacobian matrix \( J(\theta(t)) \) can be computed if the solution matrix \( X(t) \) of the following linear equation is solved, i.e.,

\[
J(\theta(t))X(t) = I,
\]

where \( X(t) \in \mathbb{R}^{6 \times 6} \), \( J(\theta(t)) \in \mathbb{R}^{2 \times 6} \) and unit matrix \( I \in \mathbb{R}^{6 \times 6} \). Since \( J(\theta(t)) \in \mathbb{R}^{2 \times 6} \) is not a square matrix, it is impossible to obtain its inverse matrix. Hence, we reformulated Equation (32) into the following form by multiplying the transposed matrix \( J^T(\theta(t)) \) to both sides of Equation (32), i.e.,

\[
J^T(\theta(t))J(\theta(t))X(t) = J^T(\theta(t)).
\]

Define \( A(t) = J^T(\theta(t))J(\theta(t)) \in \mathbb{R}^{6 \times 6} \) and \( B(t) = J^T(\theta(t)) \in \mathbb{R}^{6 \times 2} \), then the inverse-kinematic problem is converted into the solution to Equation (33) and can be solved by using the proposed CRLN model (3). That is to say, once the solution matrix \( X^*(t) \) is obtained by CRLN model (3), the inverse matrix can be computed and be substituted into Equation (31), and then the robot inverse-kinematic problem is solved.

To verify the superior robustness of proposed CRLN model (3), the kinematic control of a six-link Kinova manipulator problem can be achieved by using the CRLN model (3) and the TZNN model (12) to solve the Equation (33). It is worth mentioning out that the length of the links are set as \( l_1 = l_2 = l_3 = l_4 = l_5 = l_6 = 1.0 \text{m} \), and the initial joint angle state is \( \theta(0) = [\pi/3, -\pi/4, \pi/3, \pi/6, \pi/5, \pi/6]^T \). The perturbed cognitive periodic noise in the simulation is set as the square wave periodic noise.

The robot’s motion trajectories of a twist-circular path synthesized by CRLN model (3) and TZNN model (12) using a power-sigmoid activation function are presented in Fig. 12. As shown in Fig. 12 (a) and (c), obviously, the task of the end-effector is not completed when using TZNN model (12) and the tracking trajectories have significant difference with the expected path due to the interference of cognitive periodic noise. On the contrary, the tracking task is achieved so well and the tracking trajectories of CRLN model (3) coincide with the expected path excellently as shown in Fig. 12 (b) and (d). Furthermore, the tracking performance of two neural networks can be distinguished conspicuously through the end effector position errors in Fig. 13. Evidently, as can be seen from Fig. 13, the position error synthesized by CRLN model (3) can be converged to \( 4 \times 10^{-4} \) within 10 seconds, which is satisfied with the accuracy principle. While the position error of TZNN model (12) is oscillating and upper bound of position error is thousands time larger than that of CRLN model (3). Hence, the simulation results verify the superiority of proposed CRLN model (3) for solving the kinematic motion control problem in the disturbance of cognitive periodic noise. Besides, just as the Table VII shows, the computation time of TZNN in ten repetitive experiments is larger than the task execution time,

![Fig. 12. Comparisons of tracking trajectories and the simulation results generated by CRLN and TZNN for a Kinova manipulator tracking two-twist circle path in the presence of random periodic noise.](image_url)
Computation time cost of the online solution $X(t)$ can always converge to zero by using CRLN model (3) can be further improved by designing different kinds of activation functions and setting a permitted larger values of design parameters. Finally, a robot motion planning example has further verified the effectiveness, strong robustness and practicability of the proposed CRLN model.

VI. CONCLUSION

In this paper, a circadian rhythms learning network (CRLN) has been proposed to resist the cognitive periodic noise in time-varying dynamic system. Strict mathematical proof and theoretical analysis have been given in detail, which has shown that the computational error of the online solution $X(t)$ can always converge to zero by using CRLN model (3) with excellent convergence property. Computer simulations have verified the accuracy and efficiency of the proposed CRLN model (3). In addition, comparisons with the traditional zeroing neural network have shown the faster convergence rate and smaller convergence errors of the proposed CRLN model (3). What is more, detailed data results have demonstrated that


table reference

**Table VII**

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which means the task execution movement is finished while the computation process is still operating. On the contrary, the computation time cost of CRLN in ten repetitive experiments is less than the task execution time cost, which indicates the computation process can be finished within the expected path tracking task time cost, which is suitable for practical implementation.

In summary, the computer simulation results of a six-link planar robot manipulator kinematic motion control demonstrates the high accuracy and effectiveness of proposed CRLN model (3) for resisting the cognitive periodic noise in the time-varying dynamic system i.e., kinematics motion control problem.

REFERENCES


Zhijun Zhang (M’12) received the Ph.D. degree from Sun Yat-sen University, Guangzhou, China, in 2012. He was a Post-Doctoral Research Fellow with the Institute for Media Innovation in Nanyang Technological University, Singapore in 2013-2015. Since 2015, he has been an Associate Professor with the School of Automation Science and Engineering, South China University of Technology, Guangzhou, China. His current research interests include neural networks, robotics, and human-robot interaction.
XianZhi Deng has been with School of Automation Science and Engineering, South China University of Technology, Guangzhou, China since 2016. His current research interests include neural network, machine learning and robotics.

Lingdong Kong (S’18) has been with School of Automation Science and Engineering, South China University of Technology, Guangzhou, China since 2015. His major is Intelligence Science and Technology. His current research interests include automatic control, system engineering, and robotics.

Shuai Li (SM’14) the B.E. degree in precision mechanical engineering from Hefei University of Technology, Hefei, China, in 2005, the M.E. degree in automatic control engineering from University of Science and Technology of China, Hefei, China, in 2008, and the Ph.D. degree in electrical and computer engineering from Stevens Institute of Technology, Hoboken, NJ, USA, in 2014. He is currently with Hong Kong Polytechnic University, Hung Horn, Kowloon, Hong Kong, China. His current research interests include dynamic neural networks, wireless sensor networks, robotic networks, machine learning, and other dynamic problems defined on a graph.